Boone Tison

Homework 4.2

#5:

If a and b are any odd integers, then a2 + b2 is even.

Let a and b be any odd integers. By the definition of odd, a = 2x + 1 and b = 2y + 1. Then, a2 + b2 = (2x + 1)2 + (2y + 1)2 = 4x2 + 4y2 + 4x + 4y + 2. Let z equal 4x2 + 4y2 + 4x + 4y + 2 and factor out a 2, z is an integer because the products and sums of squares of integers are integers. Therefore, a2 + b2 = 2z, which means a2 + b2 is even by the definition of even, n = 2k.

#8:

For any integers m and n, if m is even and n is odd then 5m + 3n is odd.

Let m be any even integer and n be any odd integer. By the definition of even and odd, m = 2x and n = 2y + 1. Then, 5m + 3n = 5(2x) + 3(2y + 1) = 10x + 6y + 3. Let z = 10x + 6y + 3 and factor out a 2, z is an integer because the products and sums of integers are integers. Therefore, 5m + 3n = 2z + 1, which means 5m + 3n is odd by the definition of odd, n = 2k + 1.

#27:

The difference of any two odd integers is even. This statement is true.

Let m and n be any odd integers. By the definition of odd, m = 2x + 1 and n = 2y + 1. Then, m – n = 2x + 1 – 2y + 1 = 2x – 2y = 2(x – y) and x – y is an integer. So, m – n is even by the definition of even, n = 2k.